Multicriteria Parametric Identification of a Mathematical Model of Metal Cutting Machine's Main Drive

Yuliyan A. Angelov

Abstract—**A general multicriteria parametric identification problem is formulated for a mathematical model of a metal cutting machine main drive with linear characteristics of the elastic and dissipative properties as a nonlinear optimization problem. An optimization procedure determining the unique Pareto-optimal solution by means of direct approach and a compromising scheme based on the utopical point in the criteria space is used to find an approximate solution to the formulated problem. The suggested approach is illustrated through a test identification problem.**

Index Terms— **mathematical model, metal cutting machines, parametric identification, Pareto-optimal solutions.**

I. Introduction

The main drive (MD) is a principal unit of metal cutting machines (MCM) which determines the quality of the technological operations carried out. A basic quality criterion for the main drive performance under dynamic load is its vibration resistance. The vibration resistance can be assessed with the help of the amplitude-frequency characteristics (AFC) of the examined unit. Optimization based on this criterion is possible with a simulation model in which the main drive structure is presented by an adequate mathematical model (MM). For an existing prototype and when preliminary experimental defining of some principal characteristics of the mechanical system is possible, MM building is brought to the formulation of a parametric identification problem.

The degree of MM adequacy is largely dependent on the formulated identification problem and the method which is used to solve it. It has been quite common to formulate and examine single criterion identification problems for mechanical systems disturbed by various external influences in a certain frequency range, using the Laplace and Fournier transformations [4].

The assessment criterion for the discrepancy between experimental and simulated data in a frequency range usually is

$$I = \frac{1}{N}\sum_{j=1}^{N}\left[E\left(i\omega_j\right)\right]^2 \Omega\left(\omega_j\right) \qquad (1)$$

where N is the number of excited frequencies; $E(iw_j)$- the summary error; $\Omega(w_j)>0$ – the weighting function, determining the relative importance of the input data; $i = \sqrt{-1}$; w_j – the disturbance influence frequency. Such simplification of the formula (1) conceals the complicated and very often controversial problem of determining the weighting function $\Omega(w_j)$; thus 'avoiding' the multicriterial considerations at the expense of the using additional subjective information.

The main fault of single criterion identification methods is their limited use in MM for systems with high discretization. In this case the identified parameters may turn out to have values close to zero, which leads to inadequate conditionality of the matrices and destabilizes the calculation process.

In actual fact, identification problems are multicriteria problems [9]. A universal MOVI-method for multicriteria identification is suggested in [8], it is based on quasi-uniform probing of multidimensional parametric areas by means of the so called PSI-method (Parametric Space Investigation) [7], and selection of a set of approximately favourable solutions satisfying the Pareto-optimization principle [2].

The paper formulates a general problem for multicriteria parametric identification of MM of the MD of a MCM with linear characteristics of the elastic and dissipative properties. The problem is solved with the help of part of the calculation technology suggested in [1], which uses the PSI-method.

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II. SIMULATION MODEL

A. *Mathematical model*

Simulation is performed for steady-state operating modes of the MD with the assumptions to get a discrete mechanical system with n degrees of freedom and with linear characteristics of the elastic-dissipative ties. When certain power conditions are satisfied [3], the MD is presented by an adapted dynamic model (fig. 1) with parameters the adapted values of: the mass moments of inertia J_i of the concentrated masses; the elasticity coefficients k_i and the damping coefficients h_i; the external influence moments M_i.

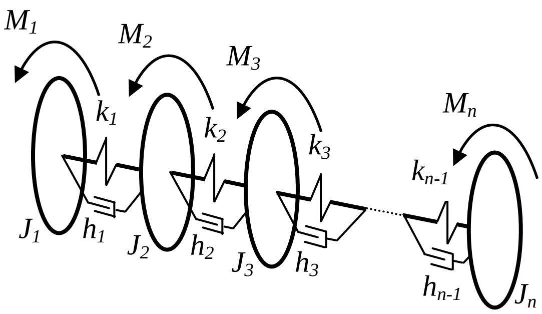

Fig. 1. Adapted dynamic model of MD of a MCM

For the case of a dissipative mechanical system with harmonic disturbances and generalized coordinates, φi rotation angles of the adjacent concentrated masses, a semi-determined mechanical system is formed and it is described by a set of differential equations

$$\mathrm{A}\varphi_i'' + \mathrm{B}\varphi_i' + \mathrm{C}\varphi_i = \mathrm{Q}, \qquad (2)$$

where: $\varphi = \{\varphi_i, i\in\mathbf{I}:=[1:(n-1)]\}$ is the generalized coordinates vector; $(\,'\,) = d/dt$; A, B and C are square matrices containing respectively the generalized inertia a_{ij}, resistance b_{ij} and elastic c_{ij} coefficients; Q is a vector with elements the generalized amplitudes q_j of the external influence. The generalized values a_{ij}, b_{ji}, c_{ji} and q_j from equation (2) are expressed by the parameters J_i, k_i, h_i, M_i of the adapted dynamic model (fig. 1).

The mathematical model simulating the AFC of the MD structure results from the solution of model (2) for the forced vibrations of the mechanical system; their amplitudes d_{ik}, in a complex form are expressed by the equation

$$\mathrm{D}=[(\mathrm{C}- f_k^{\,2}\mathrm{A})+i\, f_k\, \mathrm{B}]\backslash \mathrm{Q}, \qquad (3)$$

where $\mathrm{F}=\{ f_k, k\in\mathbf{K}:=[1: l\,]\}$ is the vector of harmonic interference frequencies.

B. *Parameterization of the mathematical model*

The mathematical model (3) is parametric manageable. To the experimentally determined on the generalized coordinate φi AFC corresponds the simulated d_{ik} in the "i" order of the matrix D with given amplitudes $p=\{H_j, j\in\mathbf{J}:=[1:n]\}$ of the harmonic interferences Mi and adjustment of the manageable parameters $u=\{J_j, h_i, k_i, i\in\mathbf{I}, j\in\mathbf{J}\}$ of the adapted dynamic model.

After introducing certain interval limitations for the parametric vector elements u, the mathematic model (3) can be generalized as

$$\begin{aligned}&\Psi(d(f), u, p)=0,\\ &u\in\mathbf{U}:=\{u\in\mathbf{E}^{3n-2}: u^- \le u \le u^+\},\\ &p\in\mathbf{P},\quad f\in\mathbf{F}=[0, f_l],\end{aligned} \qquad (4)$$

where: u^-, u^+ are limit values of the vector u; f_l is a fixed value from the frequency space.

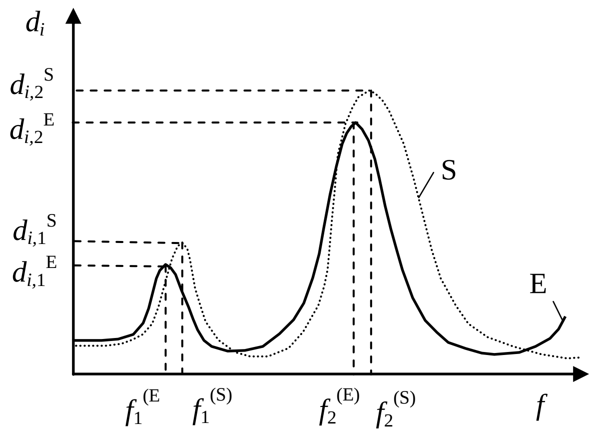

Fig. 2. Amplitude frequency characteristics:
E – experimental; S – simulated

C. *Adequacy criteria*

The degree of correspondence between the experimental "E" and the simulated "S" AFC on the generalized coordinate φi (fig. 2), is assessed with the help of two sets of criteria. Private criteria are given in a non-dimensional form in order to ensure equipollence of the assessment of the two separate sets.

The relative differences between the experimental $f_\nu^{\,\mathrm{E}}$ and the simulated $f_\nu^{\,\mathrm{S}}$ resonance frequencies form the set

$$\Phi_\nu^{\,\mathrm{I}}=\left| f_\nu^{\,\mathrm{E}}-f_\nu^{\,\mathrm{S}}\right| / f_\nu^{\,\mathrm{E}}, \quad \nu\in\mathrm{N}:=[1:(n-1)]. \qquad (5)$$

The discrepancies between the values of the resonance amplitudes of $d_{i,\nu}^{\,\mathrm{E}}$ and $d_{i,\nu}^{\,\mathrm{S}}$ form the set

$$\Phi_\nu^{\,\mathrm{II}}=\left| d_{i,\nu}^{\,\mathrm{E}}-d_{i,\nu}^{\,\mathrm{S}}\right| / d_{i,\nu}^{\,\mathrm{E}}, \quad \nu\in\mathrm{N}. \qquad (6)$$

With the formulated criteria (5) and (6), the vector criterion for the adequacy assessment of the mathematical model is

$$\Phi(u)\in\mathbf{K}:=\{\Phi_\nu^{\,\mathrm{I}}, \Phi_\nu^{\,\mathrm{II}}, \quad \nu\in\mathrm{N},\}. \qquad (7)$$

D. *Identification problem*

A vector identification problem is formulated with the help of dependencies (4) and (7)

$$u^{*}=\mathrm{opt}_{u}\{\Phi(u)\},\ \Psi(d(f),u,p)=0, \qquad (8)$$
$$\Phi(u)\in\mathbf{K},\ u\in\mathbf{U},\ f\in\mathbf{F},\ p\in\mathbf{P}.$$

where "opt" is an operator for simultaneous minimization of the private criteria $\Phi(u)$, according to the Pareto principle.

The solution to problem (8) is, in fact, finding such an admissible control vector $u^{*}\in\mathbf{U}$, which minimizes the vector criterion (7) in the sense stated above. In the criteria space this solution is a set of Pareto-optimal points, which are incomparable with each other. The choice of one compromising solution requires scaling of criterion (7) with the help of a chosen compromising scheme.

The possibility for a steady approximation of the private criteria to their ideal values can be determined with the suggested in [5] compromising scheme, which is based on the concept of 'utopic' point in the criteria space. For the formulated (7) criteria space **K**, the 'utopic' point coincides with the origin of coordinates and ($\Phi^{o}\equiv 0$) the generalized criterion takes the form

$$F^{S}(u)=\|\ \Phi(u)\ \|, \qquad (9)$$

where $\|\bullet\|$ is Euclidean norm in **K**.

III. Control Example

The potential of the suggested approach is illustrated with the control identification problem. The experimental data comprises results from an example examined in [10]. The damping coefficients values are an extent higher than the data used in the example, in order to achieve better visualization of the results from the problem which is solved.

To carry out the calculations according to the PSI-method [6], procedures for probing of the parameter space and for the selection of a set of approximately favourable solutions with the help of a simulation model (4), (7) and (9) in the Matlab program system were compiled.

The multicriteria identification approach follows N_S=1024 Sobolev sample points from the 13-dimensional parameter space with a fixed vector of harmonic interference amplitudes p=(65, 0, 0, 0, 0,)N.m.

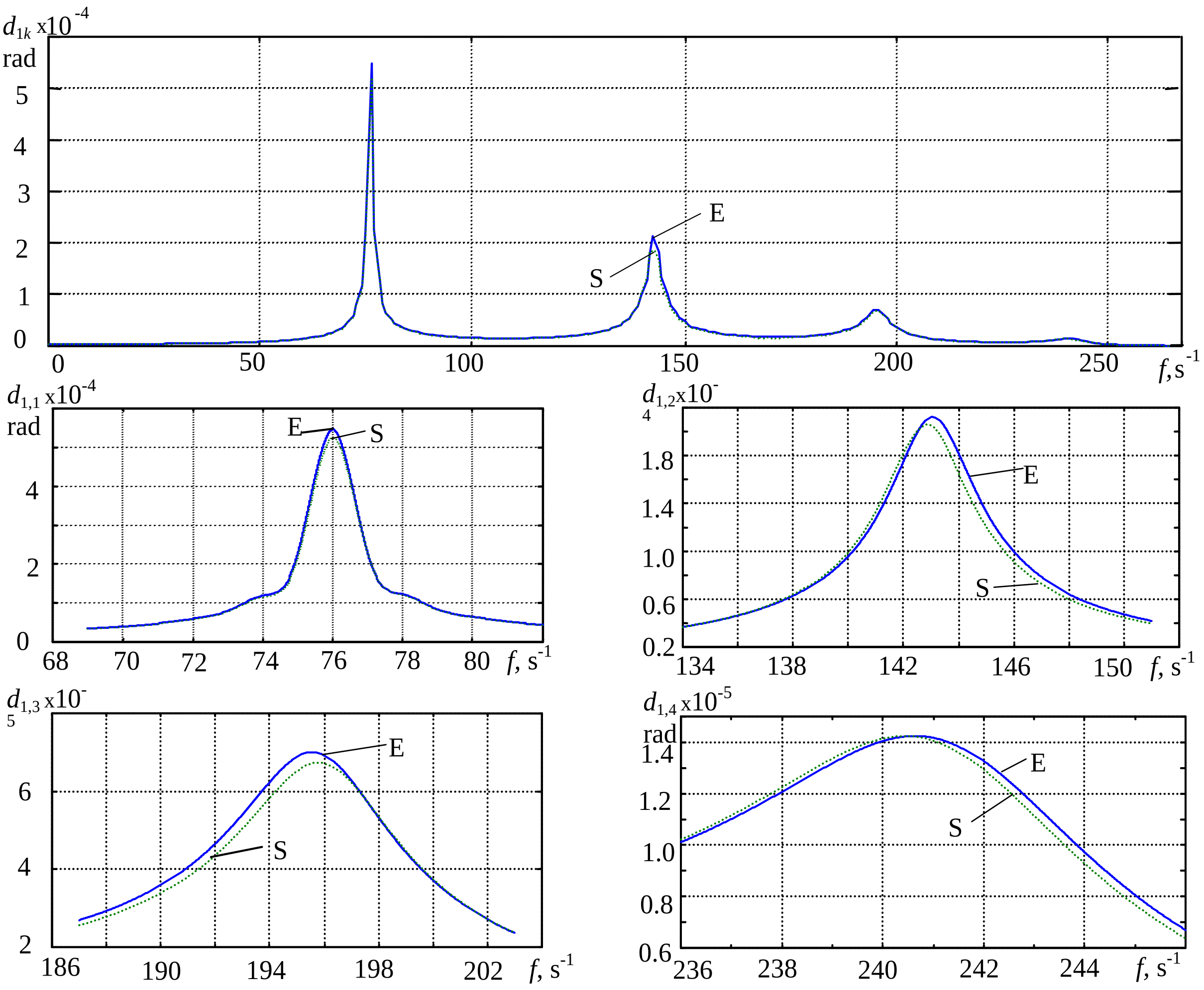

Fig. 3. Experimental "E" and identified "S" amplitude frequency characteristics.

An optimal solution according to the generalized criterion $[u^*, F^S(u^*)]$=0.0787 is found. The degrees of compromise for the private criteria from set I and set II are shown in Table I, and the respective values of the u^*vector are shown in Table II.

TABLE I
THE DEGREES OF COMPROMISE FOR THE PRIVATE CRITERIA

$\Phi(u^*)$	ν			
	1	2	3	4
$\Phi_\nu^{I}(u^*)$	0.0003	0.0016	0.0011	0.0009
$\Phi_\nu^{II}(u^*)$	0.0361	0.0283	0.0385	0.0002

TABLE II
VALUES OF THE U* VECTOR

u^*	i , j				
	1	2	3	4	5
$J_j^* \times 10^2$, kg.m^2	2,49	2,0	1,60	1,26	0,98
$h_i^* \times 10^2$, N.m.s	4,96	4,13	2,98	1,96	–
$k_i^* \times 10^6$, N.m	2,57	2,41	2,27	2,22	–

The proximity which is achieved between the experimental and the simulation data is shown in (fig.3).

REFERENCES

[1] Cheshankov, B., V. Vitliemov, P. Koev. Multicriteria parameter synthesis of mechanical systems: I Optimization approach. Machine mechanics, vol.8 (32), №4, 97-102, 2000 (in Bulgarian).

[2] Miettinen, K.M. Nonlinear multiobjective optimization. Kluver Academic, Boston, 1999.

[3] Minchev, N, V. Zhivkov, K. Enchev, P. Stoyanov. Theory of mechanisms and machines, Tehnika, Sofia,1991. (in Bulgarian)

[4] Redko, S.F., V.F. Ushalov, V.P. Yakovlev. Identification of mechanic systems. Naukova dumka, Kiev,1985.

[5] Salukvadze, M. E. An Approach to the solution on the vector optimization problem of dinamic systems. Journal of theory and applications, 38, 1982, p 409-422.

[6] Statnikov, R.B, J. B. Matusov. Multicriteria optimization and engineering. Chapman & Hall, New York 1995.

[7] Statnikov, R.B., Matusov, J.B. Multicriteria analysis in engineering. Kluwer, Dordrecht, 2002.

[8] Statnikov, R.B. Multicriteria Design: Optimization and Identification. Kluwer Academic Dordrecht, 1999.

[9] Tsonev, S., V. Vitliemov, P. Koev. Optimisation methods. Ruse, 2004, (in Bulgarian).

[10] Yordanov, Y.T. Theoretical Mechanics, Gabrovo, 1990 (in Bulgarian).